No. 122 MINOR PLANETS. I. THE ROTATION OF VESTA*

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ABSTRACT

A set of 13 light curves for 4 Vesta was obtained between November 1958 and May 1959. The set is compared with light curves observed in 1950 (by H. L. Johnson), in 1952 (D. L. Harris), in 1954 (L. Binnendijk), and 1967 (this paper). The speed as well as the sense of rotation are derived from the effect of aspect change on the apparent rotational period ("photometric astrometry"). The rotation is direct, the true period is $5^h20^m31^s.665$ ($\pm0^s.003$ p.e.), and the ecliptic longitude and latitude of the north pole are 126° (±5°) and +65° (±4°). Vesta differs from the other asteriods in that its light curve has only one maximum and one minimum, indicating a nearly spherical shape for Vesta with a darker, and slightly bluer, region on one side. The brightness-phase function shows a nonlinear increase near opposition ("opposition effect"). The colors B−V and U−B increase with the orbital phase angle (reddening with phase).

I. Observations

PHOTOELECTRIC observations of Vesta have been made by Hall (1940), Giclas (1951), Stephenson (1951), Cuffey (1953), Johnson and Harris (Groeneveld and Kuiper 1954), Binnendijk (unpublished), Haupt (1958), and Chang and Chang (1962). The previous work is inconclusive, especially as to whether Vesta's light curve has one or two maxima. Therefore, Vesta was observed during 13 nights in 1958/59, and an additional light curve was obtained in 1967. The sense of rotation and the true period of rotation are determined in Secs. II–IV. In Sec. V, the light curves are intercompared in order to obtain the brightness–phase relations at three wavelengths.

The observations were made with the 91-cm Cassegrain reflector at the McDonald Observatory, 5 November 1958–10 May 1959. In addition the McDonald 208-cm reflector was used on 2 November and 24 January. One more light curve was obtained, on 10 April 1967, with the 53-cm Catalina reflector situated near Tucson, Arizona.

A standard *UBV* photometer is used with an RCA 1P21 photomultiplier that is cooled with dry ice. The light curves are made, with the *V* filter, by comparison with a nearby star about once every 6 min. The calibration is made at least three times per light curve with a "*UBV* transfer," consisting of observations of a red and a blue *UBV* standard star. The light-curve intensities are read from a strip chart or, more recently, from the printout of a system with a digital voltmeter and papertape punch (shown in Fig. 1 of Coyne and Gehrels 1967). Extensive use is made of the computer equipment of the Numerical Analysis Laboratory at the University of Arizona. Further details of the observations and reductions have been described by Groeneveld and Kuiper (1954) and by Gehrels and Owings (1962); the present series of papers is a sequel to that work, published in this Journal, which has the

TABLE I. Summary of observations of 4 Vesta.

Obs. date 1958/59 UT	Mean *V* (mag)	*B*−*V* (mag)	*U*−*B* (mag)	Mean *V* corr. to unit dist. (mag)	Figure
2 Nov.	7.880:	···	···	4.045:	1
5 Nov.	7.847	+0.793	+0.503	4.050	2
	± .002	± .0014	± .0017	± .002	
11 Dec.	7.211	.795	.491	3.933	3
19 Jan.	6.354	.771	.458	3.489	4
24 Jan.	6.232	.763	.455	3.384	5
26 Jan.	6.211	.755:	.446:	3.369	6
28 Jan.	6.227	.761	.452	3.387	7, 8
31 Jan.	6.290	.766	.453	3.452	9
5 Feb.	6.407	.768	.468	3.562	10
20 Feb.	6.695	.783	.492	3.777	11
25 Feb.	6.780	.786	.495	3.823	12
6 May	7.809	.802	.522	4.087	13
10 May	7.852:	+0.802	+0.518	4.092:	14

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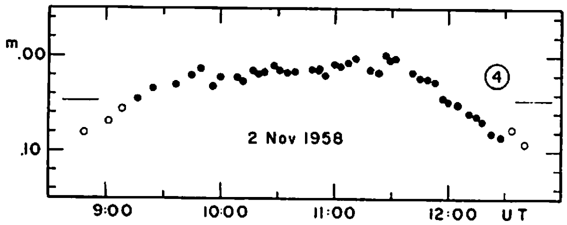


FIG. 1. Observations at −23° phase.

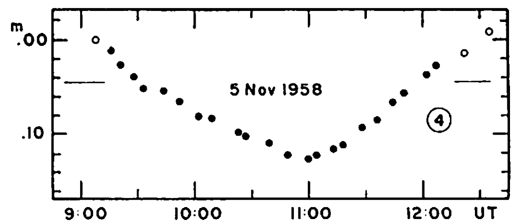


FIG. 2. Observations at −23° phase.

same format as that of the *Communications of the Lunar and Planetary Laboratory*.

Table I gives some details of the observations on Vesta. "Mean *V*" is the visual magnitude that has equal areas—over one cycle—enclosed by the light curve above and below the line of "Mean *V*." This is

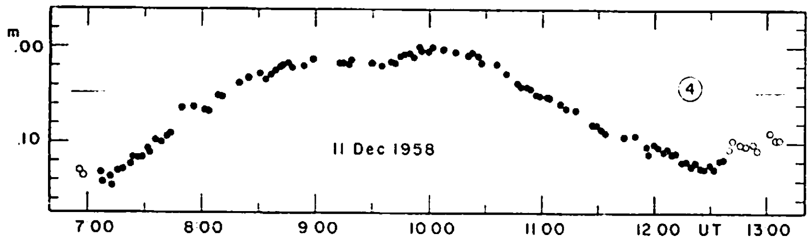


FIG. 3. Observations at −19° phase.

TABLE II. Aspect data for Vesta.

Obs. date UT	Phase angle	Distance from sun (a.u.)	Distance from earth (a.u.)	Light time	R.A. 1959	Dec. 1959	Ecliptic Long. 1950	Ecliptic Lat. 1950	Phase shift
58.11.02	−23°05	2.533	2.3086	$0^{\mathrm{d}}013$ 332	$8^{\mathrm{h}}42^{\mathrm{m}}7$	+18°24′	128°06	+ 0°17	−12°25
	± .03	±.001	±.0001	±.000 001	±.1	± 1	±.03	± .02	± .04
58.11.05	−23.00	2.532	2.2687	13 102	8 45.5	+18 19	128.73	0.27	−12.20
58.12.11	−18.51	2.511	1.8023	10 408	9 03.5	+18 42	132.72	1.76	− 8.62
59.01.19	− 3.77	2.484	1.5067	8 700	8 42.4	+22 23	126.98	4.02	− 0.54
59.01.24	− 1.94	2.480	1.4963	8 641	8 37.4	+22 57	125.72	4.27	− 0.09
59.01.26	1.73	2.478	1.4943	8 630	8 35.5	+23 10	125.20	4.36	...
59.01.28	+ 2.10	2.477	1.4930	8 622	8 33.1	+23 25	124.63	4.45	+ 0.11
59.01.31	+ 3.19	2.474	1.4936	8 626	8 30.0	+23 44	123.86	4.69	+ 0.36
59.02.05	+ 5.46	2.471	1.5000	8 663	8 24.7	+24 14	122.55	4.79	+ 1.13
59.02.20	+12.22	2.458	1.5599	9 008	8 11.2	+25 24	119.29	5.24	+ 4.48
59.02.25	+14.22	2.454	1.5917	9 192	8 07.9	+25 39	118.51	5.32	+ 5.68
59.05.06	+24.65	2.390	2.3245	13 424	8 39.3	+23 58	125.86	5.36	+13.65
59.05.10	+24.67	2.386	2.3685	13 678	8 44.2	+23 38	127.04	5.32	+13.66
67.04.10	−17.93	2.178	1.3259	0.007 657	16 01.5	−10 09	240.22	+10.29	− 8.21

TABLE III. Comparison stars and quality of nights.

Obs. date 1958/59 UT	R.A. 1959	Dec. 1959	*V* (mag)	*B−V* (mag)	Scatter of comp. readings (mag)	Remarks
2 Nov.	$8^{\mathrm{h}}42^{\mathrm{m}}8$	+18°30′	9.866	+0.945	±0.004	
	±.2	± 3	±.002	± .002		
2 Nov.	8 43.0	18 30	9.000	0.412	...	
5 Nov.	8 45.7	18 12	9.229	0.455	.003	
11 Dec.	9 03.2	18 54	8.736	1.572	.004	
19 Jan.	8 44.2	22 29	8.198	0.235	.003	Cloudy
24 Jan.	8 36.7	22 57	9.227:	1.015:	.003:	Cloudy
26 Jan.	8 35.7	23 15	9.879	1.310:	.005	Stopped by clouds
28 Jan.	8 32.2	23 20	9.830	0.490	.002	a
31 Jan.	8 30.0	23 54	8.625	0.514:	.005	Clouds about
5 Feb.	8 24.6	24 06	10.339	0.489	.005	
20 Feb.	8 10.8	25 24	9.190	1.055	.003	
25 Feb.	8 08.0	25 38	5.718	0.823	.002	
6 May	8 38.9	24 08	10.370	0.960	.003	
10 May	8 42.3:	+23 38:	10.988:	+0.837	±0.003	Clouds about

[a] Asteroid passed a 12th mag star, 2:50–4:00 UT.

determined by planimetry for the light curve of 28 January 1959, and the position of "Mean V" is given by the horizontal thin line in Fig. 7. Next, with the epochs of Table X (see Sec. V) and by superposition of Fig. 7 on the shorter light curves, the position of the horizontal line is found for the other figures. The calibration is with UBV transfers to standards in the Praesepe cluster (Table I of Gehrels and Owings 1962). No UBV calibration was made on 10 April 1967. The correction to unit distances in Table I is with 5 log$r\rho$, where r is the distance of the asteroid to the sun and ρ that to the earth (Table II). Whenever possible, probable errors are given in the tables.

Table II gives various data for Vesta at the epochs listed in the second column of Table IV. The orbital phase angle is computed with Method B of Gehrels (1957); the values differ appreciably from those in the *Ephemerides*. The last column lists F, the displacement of the center of light on the apparent disk due to the effect of phase, for 90° latitude of the pole (see Sec. III).

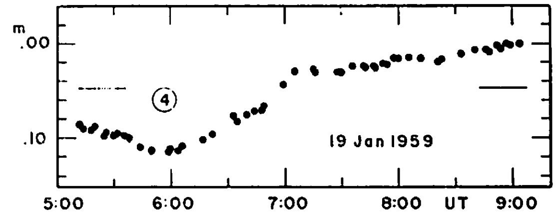
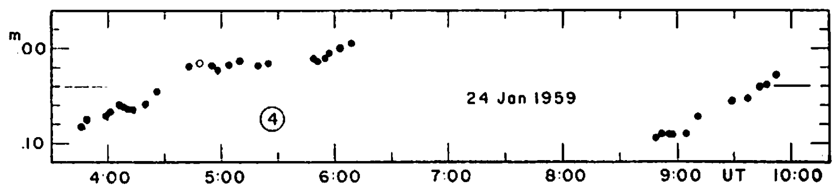


FIG. 4. Observations at −4° phase.

January (Fig. 7) has a thin vertical line that defines an epoch of maximum at $6^h47^m\!.00$ UT (uncorrected for light time). By fitting this curve onto the individual ones of other dates, the epochs of maximum light are found. This fitting of the curves had as principal requirement that the horizontal time axes be parallel, with the assumption that the comparison star is not variable and neglecting the brightness change with phase during the night.

The light curve becomes steeper at larger phase angles

FIG. 5. Observations at −2° phase.



(see Sec. V). Because of the systematic changes with phase, only *adjacent* light curves and epochs are used to derive periods of rotation.

Table III gives various data for the comparison stars. Under "Scatter of Comp. Readings," the table gives the average deviation from a smooth curve of comparison-star magnitudes versus time. Checks were made for variability of the comparison stars by plotting the results of their UBV transfers as a function of time. No conclusive evidence of variations was found, but there is some doubt in the case of the stars used on 11 December and on 10 May.

Figures 1–15 show the various light curves. The ordinates are in magnitudes, and the abscissas in Universal Time without correction for light time. Each point is the average of two, or sometimes four, integrations lasting about 30 sec each. Open circles are used when the probable error is greater than about three times that of the nightly average.

2. Sense of Rotation

Table IV gives observed epochs of maximum light in the second column. The long light curve of 28

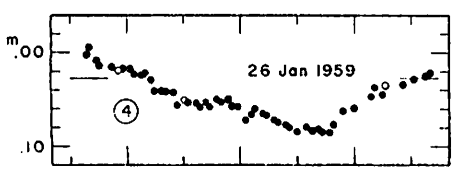


FIG. 6. Observations of Vesta made on the night of opposition, 26 January 1959.

An approximate period of rotation is derived from the light curve of 28 January; it is either 5^h20^m or 10^h41^m. With a second approximation, determined from adjacent light curves, $P_{syn}=5^h20^m\!.4$ or $10^h40^m\!.8$. The short light curves usually can be fitted to more than

TABLE IV. Observed epochs and synodic periods.

Obs. date 1958/9 UT	Observed UT	JD(c) 2436000+	Interval	No. of cycles	P_{syn}
2 Nov.	$11^h23^m\!.5$	$509^d\!.9613$			
			$3^d\!.1155$	14	$0^d\!.222$ 536
	±.5	±.0003	±.0005		
5 Nov.	14 09.4	513.0768			
			35.6201	160	626
11 Dec.	4 58.5	548.6969			
			39.1751	176	586
19 Jan.	9 08.2	587.8720			
			4.8927	22	396
24 Jan.	6 33.6	592.7647			
			1.7830	8	874
26 Jan.	1 21.1	594.5477			
			2.2263	10	631
28 Jan.	6 47.00	596.7740			
			3.1160	14	571
31 Jan.	9 34.0	599.8900			
			4.8980	22	637
5 Feb.	7 07.2	604.7880			
			15.1350	68	573
20 Feb.	10 22.1	619.9230			
			4.8981	22	643
25 Feb.	7 55.7	624.8211			
			69.9053	314	628
6 May	5 45.4	694.7264			
			4.0051:	18	0.222 504
10 May	5 53.1:	698.7315:			

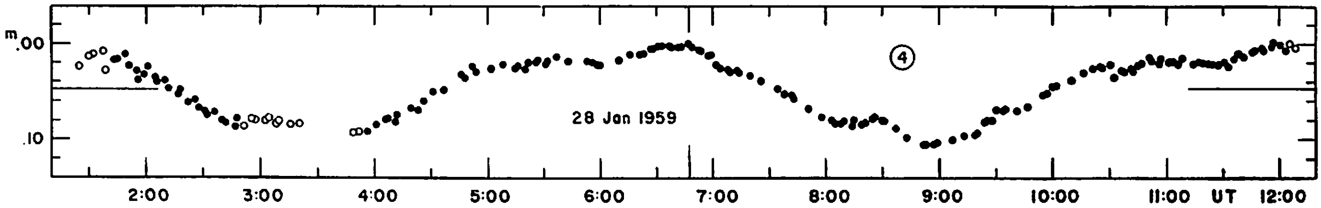


FIG. 7. Light curve of Vesta observed at +2° phase.

one part of Fig. 7, giving various epochs. One of the epochs is chosen such that there is an even number of cycles in Table IV, allowing solutions for the double period as well as for the single.

Table IV next gives the derivation of the apparent periods of rotation. The Julian day numbers are listed after correction for light time (Table II). The number of rotation cycles for each interval is found using the above period of $5^h20^m.4$. For illustration, the shorter period is adopted in Table IV, but the question of single or double period is not resolved until Sec. III.

Table V shows the effect of a change in longitude—change in aspect—on the synodic period. First, the weighted average of the periods is taken over intervals that have an increase in longitude; namely, 2 November–11 December, 5 November–11 December, 20 February–6 May, and 25 February–10 May. The weighting is done with the number of cycles. The second group in Table V, with longitude decreasing, gives the weighted average synodic period of the eight adjacent intervals between 11 December and 25 February.

The difference between the two mean periods of Table V is about 30 times the probable error. With a high confidence level, therefore, it can be stated that

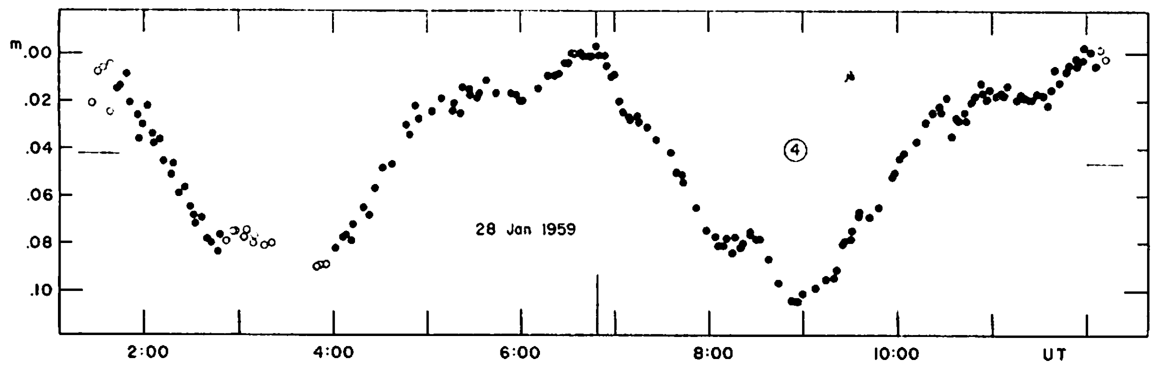


FIG. 8. As for Fig. 7, plotted on an expanded ordinate scale.

the rotation of Vesta is *direct*, which is the same sense of rotation as that of the earth, for instance. For retrograde rotation, the lower period in Table V would have been the larger one.

3. *Speed of Rotation*

Let an epoch—of maximum light, for example—be denoted by E, its ecliptic longitude by λ, the following epoch by E' at longitude λ', and the number of cycles between them by N. The number of cycles is increased for direct, and decreased for retrograde, rotation when the longitude is increasing. For direct rotation, provided β_0, the ecliptic latitude of the pole, is equal to 90°,

$$(E'-E)=\left[N+\frac{(\lambda'-\lambda)}{360}+\frac{(F'-F)}{360}\right]P_{\rm true}, \qquad (1)$$

while for retrograde rotation the two plus signs are replaced by minus signs.

In addition, there is a shift due to the change in phase. At small phase angle α, the illuminated part of the projected disk of a spherical body (unit radius)

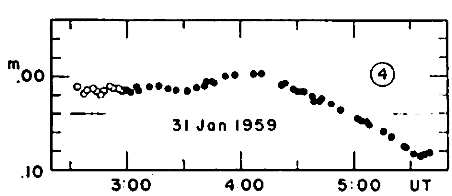


FIG. 9. Observations at +3° phase.

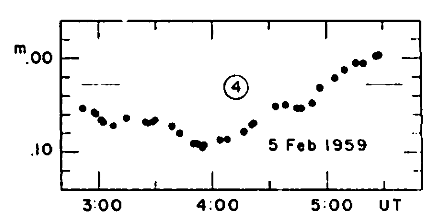


FIG. 10. Observations at +5° phase.

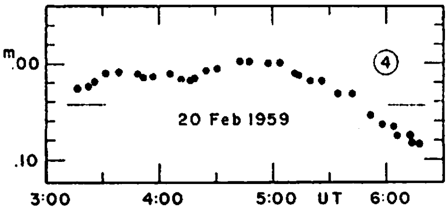


FIG. 11. Observations at +12° phase.

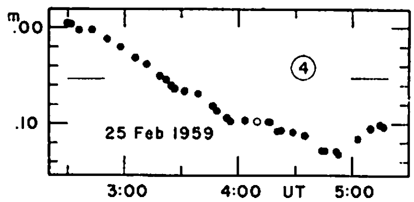


FIG. 12. Observations at +14° phase.

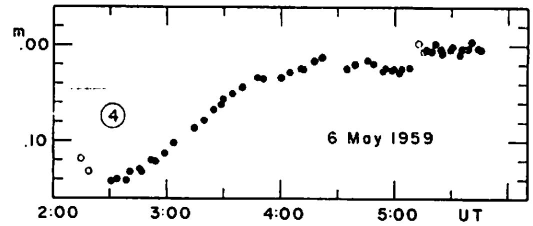
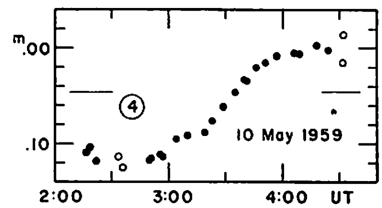


FIG. 13. Observations at +25° phase.

FIG. 14. Observations at +25° phase.



still is approximately circular with apparent radius of $1-\frac{1}{2}(1-\cos\alpha)$. For isotropic scattering, the center of light would be shifted by f degrees, where $\sin f = \frac{1}{2}(1-\cos\alpha)$. For scattering predominant in the backward direction, the center of light would be shifted by α degrees. The asteroidal scattering law is intermediate, and an intermediate function F is defined,

$$F=\{[I(0)-I(\alpha)]/I(0)\}(\alpha-f)+f. \qquad (2)$$

The relative intensities at various phase angles, $I(\alpha)$, are computed with Table XI; $I(0)$ is actually taken at 1°.5 phase. The results are listed in the last column of Table II. Before opposition the phase angle is defined with negative values and so is the phase shift in Table II.

When $\beta_0 \neq 90°$, the longitude differences, $(\lambda'-\lambda)$, are replaced by differences in L,

$$\sin L = \cos\beta \sin(\lambda-\lambda_0)/\cos B, \qquad (3)$$

$$\sin B = \sin\beta_0 \sin\beta + \cos\beta_0 \cos\beta \cos(\lambda-\lambda_0), \qquad (4)$$

where λ and β are the ecliptic longitude and latitude of observation, and λ_0 and β_0 are the ecliptic longitude and latitude of the pole. Similarly, one could derive a differential relation for the phase-shift corrections, $(F'-F)$. We actually use a mechanical coordinate converter designed by Dr. G. Van Biesbroeck.

Approximate values of λ_0 and β_0 are first obtained from a plot (Gehrels and Owings 1962) or analytic expression (Cailliatte 1956) for amplitude versus longitude of light curves observed in various years; the amplitude is the largest when the asteroid is seen equatorially and the least when observed pole-on. Next, values of λ_0 and β_0 are tried in the above expressions in order to have all intervals yield the same, true, period.

Table VI gives the rotation period computed first with expressions (1) and (2). The first line gives the weighted mean of the periods computed for the 12 intervals of Table IV, weighted with the respective number of cycles. In the second line is the weighted mean residual, without regard to sign, of the individual periods with respect to the mean of the first line. Under "Direct Rotation," the analysis is made with the plus signs in expression (1), and with minus signs for the last two columns. Under "single" the computation is made with the number of cycles shown in Table IV, while under "double" half that number is used.

There are some indications that Vesta has the period of about 0^d.2226, or 5^{h}20^m, with single maximum and minimum. In the light curves of Figs. 1–14 no systematic difference between the one maximum (or minimum) and another can be found. Furthermore, the time difference between consecutive maxima and

FIG. 15. Observations made in 1967.

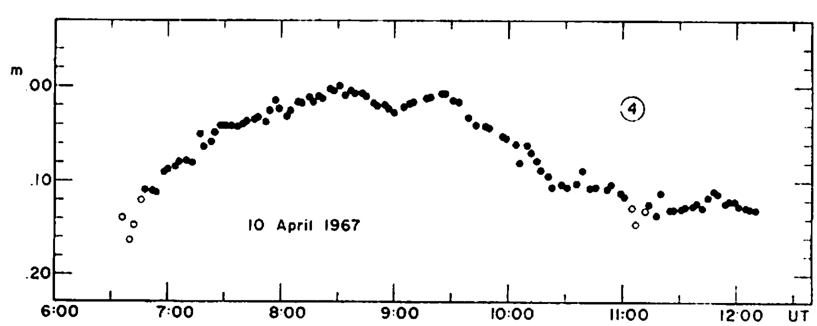


TABLE V. Period changes with longitude.

When the longitude	No. of cycles used	No. of epochs used	$\bar{P}_{syn}$	Difference
Increases	501	7	0.222 624	0.000 038
Decreases	342	9	0.222 586	±1

minima appeared to be about 5^h20^m, at all times that this could be checked. The principal criterion, however, is the extent to which expressions (1) and (2) describe the importance of the longitude and phase corrections relative to the speed of rotation. The residual, $|\bar{\Delta}|$ in Table VI, is the smallest for the first of the four cases. $\bar{P}_{sid,incr}$ and $\bar{P}_{sid,decr}$ are determined by groupings exactly as for the two periods of Table V. It is seen that the periods are about the same only in the first case. In the other three cases, expressions (1) and (2) did not describe the physical situation as the two periods differ by amounts much larger than the precision ($\pm 0^d000$ 001). It is of interest to compare each value of $\bar{P}_{sid,incr}$ and $\bar{P}_{sid,decr}$ with its value of $\bar{P}_{sid}$ (first line of Table VI) in order to see the overcorrection or lack of correction. It is noted, however, that even in the first case the two periods differ appreciably. In the lower part of Table VI, therefore, the computations are repeated with expressions (3) and (4) using the longitude and latitude of the pole of expressions (5) and (6).

In summary, Vesta's light curve has the shorter period. Additional proof is found with Table VII, below, where the observations of various years can only be fitted together with the short period and direct rotation.

4. *Orientation of the Pole*

Table VII gives the comparison of observations made at seven different longitudes. Johnson's light curve, at longitude 36°.33 and latitude −7°.55, was superposed on the present Fig. 8; the epoch of maximum defined at $6^h47^m.00$ of 28 January 1959 UT is found to occur at $3^h26^m.0$ ($\pm 0^m.9$ p.e.) of 22 December 1950 UT. Incidentally, an error was found in the original records for Fig. 1 of Groeneveld and Kuiper (1954): a point plotted at $3^h30^m.4$ UT was actually observed at $3^h26^m.0$ UT. The "Interval" in the second column of Table VII is listed after the usual correction for light time and a nearly negligible correction to Ephemeris Time. Similarly, by superposition of the original light curve worksheets, the epoch of maximum is found to occur at $9^h10^m.5$ ($\pm 0^m.4$) of 7 March 1952 UT, and $9^h25^m.2$ ($\pm 0^m.9$) of 10 April 1967 UT. The 1952 light curve was obtained by Harris (Fig. 2 of Groeneveld and Kuiper 1954) at longitude 155°.26 and latitude +10°.15.

I am indebted to Dr. L. Binnendijk for sending his unpublished observations made on 21 December 1954, at the Cook Observatory in Pennsylvania. The longitude (1950) is 82°.90, the latitude −3°.70, and the epoch of maximum is found to occur at 2^h59^m ($\pm 2^m$) UT. The combinations of epochs in Table VII are selected such that the phase corrections are at a minimum. In the third interval, however, there is an

TABLE VI. Attempts at finding the rate and direction of rotation.

	Direct rotation single	Direct rotation double	Retrograde single	Retrograde double
$\bar{P}_{sid}$	0ᵈ222 588	0ᵈ445 140	0ᵈ222 625	0ᵈ445 285
$\lvert\bar{\Delta}\rvert$	.000 024	.000 039	.000 042	.000 108
$\bar{P}_{sid,incr}$	.222 595	.445 130	.222 654	.445 367
$\bar{P}_{sid,decr}$	.222 586	.445 171	.222 586	.445 171
$\bar{P}_{sid}$	.222 586	.445 131	.222 627	.445 295
$\lvert\bar{\Delta}\rvert$	.000 021	.000 049	.000 046	.000 124
$\bar{P}_{sid,incr}$	.222 589	.445 107	.222 660	.445 390
$\bar{P}_{sid,decr}$	0.222 587	0.445 179	0.222 584	0.445 163

TABLE VII. Period determinations, for 90° latitude of the pole.

Epochs	Interval	No. of cycles	Period
50.12.22–59.02.25	2987ᵈ1888	13420.2216	0ᵈ22258 863(±7)
52.03.07–59.01.28	2517.8997	11311.9136	831(±7)
54.12.21–67.04.10	4493.2677	20186.4137	871(±9)
58.12.11–67.04.10	3041.9665	13666.2998	0.22258 889(±5)

TABLE VIII. Period[a] determinations for various latitudes and longitudes of the pole.

Lat. β_0	Long. λ_0	For the epochs of Table VII: 1950	1952	1954	1958	$\bar{P}$	$\lvert\bar{\Delta}\rvert$
	118°	866	849	863	880	868	10
72°	128	875	848	861	876	868	10
	138	882	848	860	870	867	10
	118	863	865	857	884	872	11
62	128	880	868	854	875	872	6
	138	895	864	852	857	866	13
	118	858	886	852	894	879	16
52	128	888	896	846	869	876	14
	138	915	897	842	841	869	32
	348	863	835	871	892	871	19
72	358	864	834	871	891	870	18
	368	864	833	872	891	870	19
	338	872	843	869	890	873	15
62	348	873	841	870	885	871	13
	358	875	837	869	881	869	14
	328	872	857	865	887	874	11
52	338	888	855	863	877	873	10
	348	892	849	864	867	868	10
	318	864	868	861	909	884	22
45	328	885	873	857	882	877	7
	338	891	867	858	864	870	9
	318	885	884	858	909	892	15
40	328	898	885	853	875	879	11
	338	917	878	854	852	872	22

[a] Only the last three decimals are listed; 866, for instance, stands for $P = 0^d22258$ 866.

TABLE IX. Longitudes[a] (and latitudes) on Vesta, for five dates and two combinations of latitude and longitude of the pole.

Lat β_0	Long λ_0	50.12.22	52.03.07	54.12.21	59.01.28	67.04.10
65°	126°	94°(+ 7°)	214°(−32°)	135°(−14°)	179°(−30°)	296°(0°)
45	331	247 (−11)	5 (+34)	283 (+18)	327 (+35)	82 (−7)

[a] These are differential longitudes of the center of the hemisphere that gives the light curve maximum when it has the earth in its zenith, with respect to the longitude of the pole.

appreciable difference in phase angle between the two epochs, and a correction of $-0^{\mathrm{d}}.0010$ was made to allow for the change in the light curve with phase (shown in Fig. 16).

Chang and Chang (1962) observed Vesta in 1958 and 1961 at the Purple Mountain Observatory. These observations came to my attention after completion of this paper. The 1961 observation should be combined with a light curve to be made at $+9^{\circ}.1$ phase in 1968 (see Sec. VI).

With Table VII, an important conclusion is reached, namely, that no consistency of the periods can be obtained for the other categories of Table VI: "single, retrograde," etc. None of these attempts succeeded.

The cycles in Table VII are computed with expressions (1) and (2), and the periods, therefore, are for the case of $\beta_0=90°$. It is seen that the periods differ by amounts appreciably larger than the probable errors. Although the amplitudes of light curves observed at various longitudes differ only slightly, it is enough to indicate some deviation from $\beta_0=90°$, namely with longitude of the pole near 130° or 310°. The analytical method of Cailliatte (1956), for the relation between light-curve amplitude and aspect, was used by Chang and Chang (1962), resulting in a longitude of the pole at 303° and the latitude at +74°. The present method is independent of any assumptions regarding the shape of the asteroid and the relationship between amplitude and aspect. Whereas Table VII gives $\bar{P}=0^{\mathrm{d}}.222\ 588\ 68$ and $|\bar{\Delta}|=0^{\mathrm{d}}.000\ 000\ 19$, this residual is to be reduced to about $0^{\mathrm{d}}.000\ 000\ 05$ by the proper choice of longitude and latitude of the pole. Provided that sufficient observations are available, at the proper phase angles (see Sec. VI), the present method appears to be more precise, and it definitely resolves whether Vesta's light curve has one or two maxima.

Table VIII gives the last three significant decimals of the period computed for various trial values of λ_0 and β_0. The calculations are for the four intervals of Table VII, followed by columns for the weighted mean period and the weighted mean residual. The weights are with the reciprocals of the squares of the probable errors in the last column of Table VII.

A minimum value of $|\bar{\Delta}|$ is readily found in Table VIII, namely for $\beta_0=65°$ and $\lambda_0=126°$. There still is, however, a second possibility. Incidentally, if the latitude of observation had not been taken into account, there would have been no ±180° resolution for the longitude of the pole. The second possibility is seen in Table VIII near $\beta_0=45°$ and $\lambda_0=331°$.

Table IX shows what regions are observed on Vesta at the various epochs, for the above two possible combinations of ecliptic latitude and longitude of the pole. The "longitude on Vesta" starts from the meridian through the ecliptic longitude of the pole, and it increases in the same direction as the ecliptic longitude. With direct rotation, one observes, in a light curve, decreasing Vesta longitudes. Positive and negative Vesta latitudes are in the same direction as those in the ecliptic reference frame. It is seen in Table IX that the first solution, with $\beta_0=65°$, gives less aspect variation because the observations are somewhat closer to the Vesta ecliptic. This effect is, however, less strong than might have been expected from the large latitude difference of the two solutions; the reason for this lies in the 7° inclination of Vesta's orbit.

The values of $|\bar{\Delta}|$ in Table VIII should be considered with some reservation; they are modified if, for instance, the relative weights of the periods are adopted differently. However, the first possibility, with $\beta_0=65°$, appears more likely because of the internal consistency of the 1958/59 observations when expressions (5) and (6) are applied (lower part of Table VI). When $\beta_0=45°$ and $\lambda_0=331°$ are applied instead, the consistency is less than the probable error of $\pm 0^{\mathrm{d}}.000\ 001$ allows, with the following values in the second column of Table VI: $\bar{P}_{\mathrm{sid}}=0^{\mathrm{d}}.222\ 583$, $\bar{P}_{\mathrm{sid,incr}}=0^{\mathrm{d}}.222\ 589$, and $\bar{P}_{\mathrm{sid,decr}}=0^{\mathrm{d}}.222\ 582$.

The following values and their estimated probable errors are adopted. For the ecliptic latitude of the

TABLE X. Computed epochs and residuals.

Obs. date 1958/59 UT	Computed UT	Observed minus final solution	Computed other trial
2 Nov.	$11^{\mathrm{h}}19^{\mathrm{m}}.5$	$+4^{\mathrm{m}}.0$	$+5^{\mathrm{m}}.3$
5 Nov.	14 07.1	+2.3	+3.2
11 Dec.	4 56.0	+2.5	+3.0
19 Jan.	9 08.1	+0.1	+1.3
24 Jan.	6 38.5	−4.9	−4.4
26 Jan.	1 22.3	−1.2	−0.8
28 Jan.	6 47.00	...	...
31 Jan.	9 33.5	+0.5	−0.1
5 Feb.	7 04.9	+2.3	+1.9
20 Feb.	10 21.0	+1.1	+1.0
25 Feb.	7 53.9	+1.8	+2.5
6 May	5 41.4	+4.0	+3.5
10 May	5 52.6	+0.5	−0.2

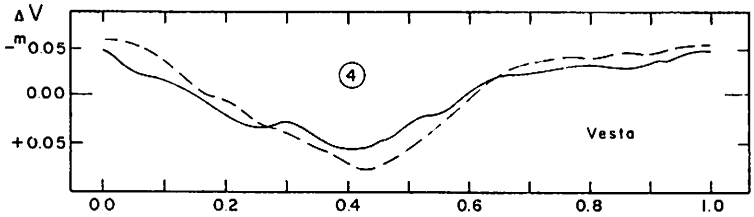


FIG. 16. The change in shape of the light curve with change of orbital phase. The *solid* line is as observed near opposition, and the *broken* line is for about 24° phase angle.

pole,

$$\beta_0 = 65° \pm 4°. \quad (5)$$

The longitude of the pole,

$$\lambda_0 = 126° \pm 5°. \quad (6)$$

The true or sidereal period of rotation is

$$0^d.222\ 588\ 71, \text{ which is } 5^h20^m31^s.665\ (\pm 0^s.003). \quad (7)$$

Table X gives an application of expressions (1)–(7), namely to compute the epoch of maximum light for the various light-curve observations in 1958/59. In the third column the differences, observed minus computed, of the epochs in Table IV are listed. The residuals are surprisingly high as compared to the probable error of $\pm 0^m.5$ in Table IV, which was based on the consistency of the fitting of light curves. Solutions were tried with a different period, with the phase corrections omitted, etc.; but very little difference is thereby made in these residuals. One of the other trials is shown with residuals in the fourth column; it was for $\beta_0 = 90°$ and period $0^d.222\ 608$. The conclusion is made from Table X that the actual precision of fitting short light curves is of the order of 1 or 2 min, rather than $\pm 0^m.5$. Superimposed on the scatter is a systematic effect due to the change in shape of the light curve with phase.

5. *Phase Functions*

Figure 16 shows how the shape of the light curve changes with phase. The "Mean V" levels of Table I were used to construct the light curves of Fig. 16. "Mean V" had been obtained from superposition of the light curves using the computed times of maximum with the residuals of the fourth column of Table X. (The final solution gave the residuals of the third column, but the differences seem trivial for the determination of "Mean V" and the work was not redone.) The solid line in Fig. 16 is for 28 January 1959, and the dashed line for 2 and 5 November 1958, and 6 May

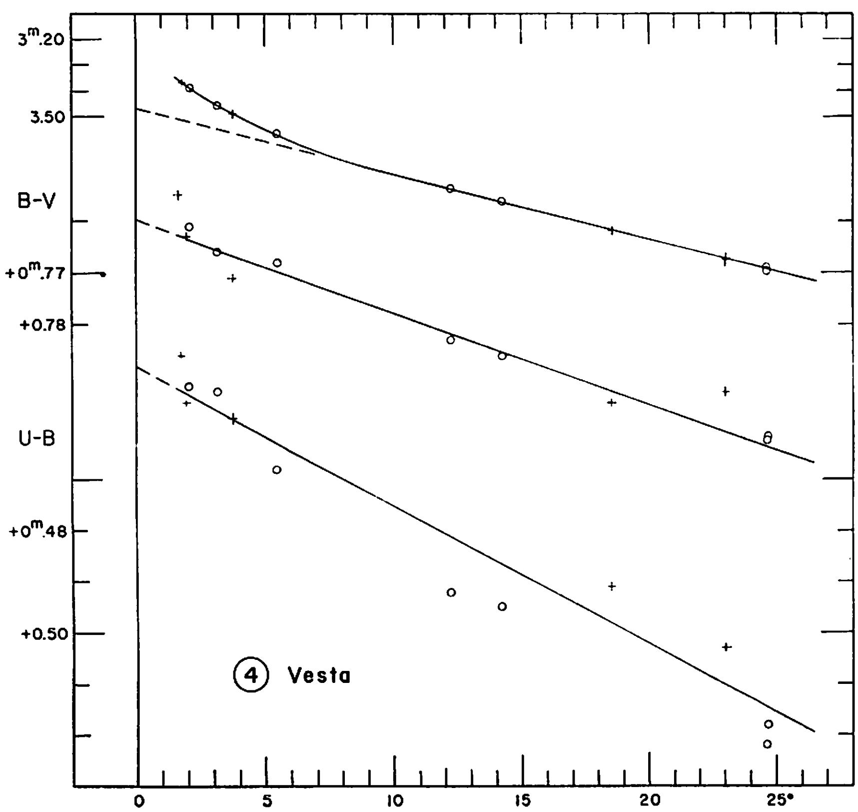


FIG. 17. Phase functions of Vesta. *Abscissas*, the orbital phase angle in degrees. *Ordinates*, *top curve*, the observed magnitudes (V on the UBV system) reduced to unit distances from the sun and earth; *middle curve*, the $B-V$ colors; *bottom curve*, the $U-B$ colors. Crosses are for observations made before and circles for after opposition.

TABLE XI. Magnitude-phase relations for Vesta, 1958/59.

Phase	V	$B-V$	$U-B$	V corr.	B corr.	U corr.
1°5	3.340	+0.758	+0.452	+0.126	+0.131	+0.127
2.0	3.382	.762	.454	+ .084	+ .085	+ .080
3.0	3.444	.766	.456	+ .022	+ .018	+ .010
4.0	3.499	.769	.459	− .033	− .039	− .050
5.0	3.542	.770	.462	− .076	− .084	− .097
6.0	3.589	.772	.464	− .123	− .132	− .148
7.0	3.630	.773	.467	− .164	− .174	− .193
8.0	3.669	+0.774	+0.470	−0.203	−0.214	−0.236

For larger phase angles, use 0.0253 mag/deg for V, 0.0264 (B), and 0.0291 (U).

1959. The amplitude changed from 0.104 mag at opposition to 0.137 mag near 24° phase. The change between November and January, and between January and May, is more than 20° in phase and only a few degrees in longitude. The conclusion is therefore made that the changes in shape are due to a change in phase rather than in aspect. A similar conclusion was reached for 20 Massalia, where there was a change in amplitude from 0.17 to 0.23 mag as the phase changed from 0° to 20° (Gehrels 1956). No difference in shape could be detected between −23° phase (November 1958) and +25° phase (May 1959), but this conclusion is based on poor evidence as the light curves were short, and May 10 may have had some trouble (see Sec. I).

Figure 17 gives, at the top, the mean brightness of each light curve as a function of the orbital phase. The "Mean V" values were used after reduction to magnitudes at unit distances from sun and earth (Table I). It is seen in Fig. 17 that the phase function is the same before as after opposition. The precision of each point is ±0.002 mag. (p.e.) which is about the width of the drawn line. At small phases the brightness increases nonlinearly ("opposition effect"). Extrapolation of the straight line gives the absolute magnitude,

$$g_V = 3.466. \tag{8}$$

Figure 17 also shows the average colors for each night (Table I) as a function of orbital phase. It is seen that the $B-V$ relation is the same before and after opposition, but that there is an appreciable difference in the $U-B$ results. The scatter of the points with respect to the mean $U-B$ line in Fig. 17 is ±0.0045 (p.e.), which is nearly three times the internal probable error of the determinations (±0.0017 mag) so that the effect probably is real. The scale of ordinates for $B-V$ and $U-B$ is 20 times that of the upper curve; 0.002 mag now is about the size of the symbols. It is also seen in Fig. 17 that there may exist an opposition effect in the color–phase relations. The most marked effect, however, is a reddening with phase. Ignoring the nonlinearity near zero phase,

$$B-V = 0.760 + 0.0018|\alpha|, \tag{9}$$

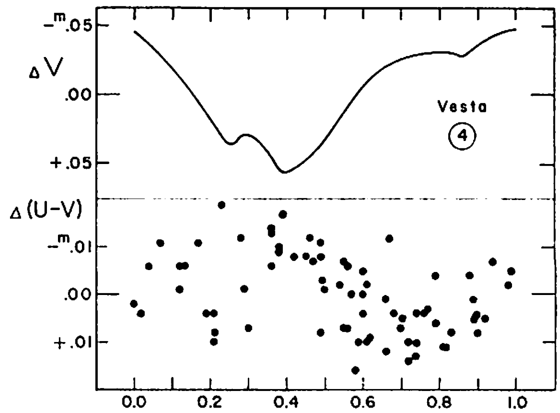


FIG. 18. Color changes over the surface of Vesta. *Abscissas*, the rotational phase. The *top curve* is a sketch of the light curve. A *dot* shows the individual color observation ($U-V$) minus the nightly mean.

and

$$U-B = 0.448 + 0.0027|\alpha|, \tag{10}$$

where α expresses the orbital phase angle in degrees.

Table XI shows the detailed phase relations in *V*isible, *B*lue, and *U*ltraviolet light. The B values were obtained with a least-squares solution of the observations at phases larger than 3°0 which gives $B-V = 0.763 + 0.0016|\alpha|$, and by drawing a small opposition effect through the points at smaller phase angles. The drawn lines of Fig. 17 were used for the V and $U-B$ values.

The last three columns of Table XI give the corrections to absolute magnitude for comparison with the corrections generally adopted for the asteroids (Gehrels 1967).

Figure 18 shows a sketch of the light curve of Vesta and at the same time the differences in color, $U-V$, from the mean. For each individual transfer—there were at least three transfers made during a light-curve run—the difference from the mean color for the night is plotted. Zero and 1.0 rotational phases are at maximum light (the epoch shown with a vertical line at 6^h47^m UT in Fig. 7). It is seen in Fig. 18 that the darker side of Vesta also is slightly bluer.

6. Concluding Remarks

Vesta appears to be a nearly spherical object with nonuniform reflectivity and color over the surface. The normal reflectivities are computed by Haupt (1958) to be of the order of 52% on one side and 45% on the other. With such high reflectivity Vesta differs appreciably from the common asteroids, that have on the average about 7% reflectivity (G. P. Kuiper, personal communication). Vesta's light curve has one maximum and one minimum, but this statement has only relative merit. The feature sketched in Fig. 18 near 0.87 rotational phase, for instance, might be called a secondary minimum. The significant conclusion is that Vesta is nearly spherical whereas most asteroids are not. Is Vesta perhaps an escaped satellite?

The opposition effect, the color being consistently redder than that of sunlight, and the reddening with phase, are phenomena of a porous surface texture with dimensions in the micron–millimeter range. The effects on Vesta are similar to those observed on the moon; some interpretations have been discussed by Gehrels, Owings, and Coffeen (1964) for the lunar surface. The cause of the porous microtexture perhaps lies in bombardment by the solar wind and subsequent tunneling and sputtering. Theoretical work appears urgently needed.

The difference between $U-B$ before opposition and after opposition (Fig. 17), if real, is not understood.

The determination of longitude and latitude of the pole can be checked and considerably improved by obtaining three light curves during the next opposition. A light curve at $+3\overset{\circ}{.}1$ phase (November 1968) will allow comparison with the observations of 1952, 1954, and 1959. A light curve at $+9\overset{\circ}{.}1$ phase will match the one of Chang and Chang (1962) made on 7 December 1961. A light curve at $+18\overset{\circ}{.}0$ phase will be ideal for comparison with the 1950 observation, especially because the aspect will be nearly the same in 1968 as it was in 1950. It is important to have the phase angles close to those mentioned so as to minimize the uncertainties in the phase correction of expression (2).

The uncertainty of the phase correction did not appreciably affect the present determination of longitude and latitude of the pole because of the selection of the epochs compared in Table VII. That expression (2) is close to the truth was found in the work for Table VI: with only the f correction the first entry in Table VI is $P_{sid}=0\overset{d}{.}222\ 606$.

The long light curve of 28 January appeared essential for the solutions of period and phase relations. At large phase angle, it would have been better to have longer light curves; especially the one of 10 May 1959 is marginal. Precise timing, using WWV signals for calibration, is essential in this work. In the future for bright objects, it will be better always to plot the mean of two integrations rather than four, in order to show fine details of the shape of the light curve. A better attempt should have been made to find comparison stars of the same color as that of the asteroid. The desirability of having two comparison stars during the night has to be weighted against the difficulty of finding two conveniently nearby stars. Extension of the wavelength range of the observations into the infrared may be found useful (see Fig. 3 of Haupt 1958).

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