No. 52 ATMOSPHERIC EXTINCTION CORRECTIONS IN THE INFRARED

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ABSTRACT

The "square-root" law and the exponential, or Beer's, law are compared in relation to their use for extinction corrections in the infrared and are examined from a theoretical standpoint. The computations indicate that more accurate extrapolations of observed intensities outside the atmosphere may be made using Beer's law than by using the square-root law.

The exact procedure that should be used for extinction corrections in the infrared part of the spectrum has been a matter of controversy. Strong (1939) and Sinton and Strong (1960) have advocated the use of the so-called "square-root" law, while others, including myself, have used the exponential, or Beer's, law. The difference of opinion has to do with the effect of the saturated and partly saturated atmospheric line and band absorption in the infrared. This matter is examined here in some detail from a theoretical standpoint.

1. Beer's Law

In general, the differential equation describing atmospheric (or other) extinction is

$$\frac{\mathrm{d}\,I(\lambda)}{I(\lambda)} = -K(\lambda)\,\mathrm{d}M, \qquad (1)$$

where $I(\lambda)$ = the intensity at any point in the atmosphere,
$K(\lambda)$ = the extinction per unit air mass,
M = the air mass.

Upon integration, we obtain:

$$\frac{I(\lambda)}{I_0(\lambda)} = \mathrm{e}^{-K(\lambda)M}, \qquad (2)$$

where $I_0(\lambda)$ is the intensity incident upon the atmosphere.

This is the exponential, or Beer's, law. It is valid for *monochromatic radiation*, regardless of the absorption mechanism. If we assume that $K(\lambda) = K$, independent of wavelength, (2) becomes

$$\frac{I}{I_0} = \mathrm{e}^{-K \cdot M}, \qquad (3)$$

the equation that has often been used for extinction corrections in the visible part of the spectrum. If, however, $K(\lambda)$ is not independent of λ, we must integrate over the pass-band of the filter, and obtain

$$\frac{I}{I_0} = \int_0^\infty \mathrm{e}^{-K(\lambda)M}\,\mathrm{d}\lambda. \qquad (4)$$

Equation (4) is true in general, regardless of the mechanism or amount of absorption.

2. Square-Root Law

It is well known that, for completely saturated lines, the equivalent width of an absorption line varies approximately as the square root of the number of absorbing particles. Therefore, the equivalent width,

$$W \propto \sqrt{M}, \qquad (5)$$

and

$$I_0 - I = Q\sqrt{M},$$

or

$$I = I_0 - Q\sqrt{M}, \qquad (6)$$

where

Q = the average effective extinction for the entire filter band at unit air mass.

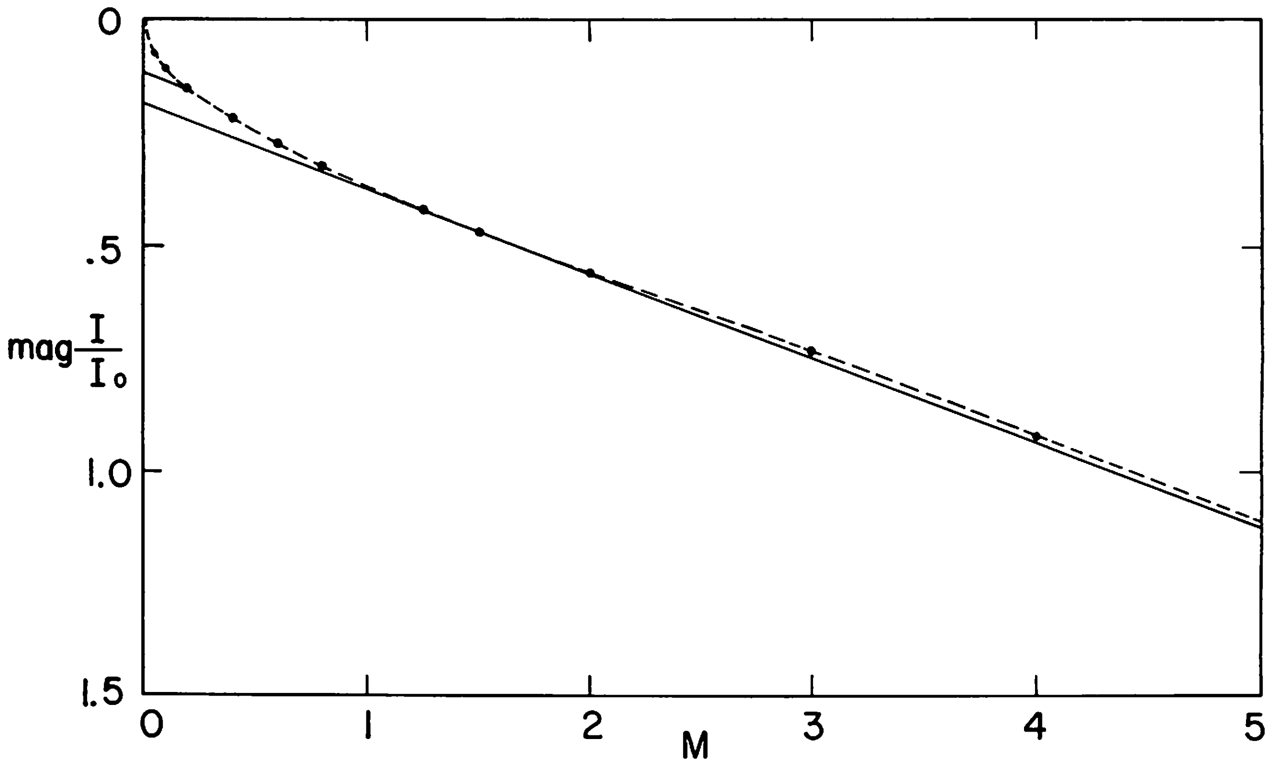

Fig. 1 Extinction versus air mass, M. The dotted line represents Equation (6), while the solid straight line represents Beer's law. It is assumed that all of the extinction is due to saturated lines and bands.

This is the justification for the square-root law of extinction correction. The derivation of Equation (6) involves the following tacit assumptions:

(1) Within the filter pass-band, the only source of absorption is nonoverlapping lines and bands;

(2) All lines and bands are completely saturated at all air masses, even in the limit as $M \rightarrow 0$.

There can be no doubt that Equation (6) is valid, subject to these conditions. On the other hand, in no practical case are these two assumptions completely valid. For example, Equation (6) has been used by Sinton and Strong to extrapolate their observations to $M = 0$, so as to obtain the outside-atmosphere energy. But, as the air mass is reduced, a point will be reached where the lines are no longer saturated, and then Equation (6) no longer applies, since for smaller air masses $W \propto M$, approximately.

3. *Comparison of Systems*

In order to illustrate the differences between the square-root and exponential laws, Figure 1 has been plotted. The dotted line represents Equation (6), while the solid straight line represents the Beer's law extrapolation of the data at $M = 1$ and 2 to outside the atmosphere. The extinction coefficient is such that $I\,(M = 1)\,/\,I\,(M = 2) = 1.2$; thus, the Beer's law extrapolation predicts about 20 percent extinction at the zenith, while Equation (6) predicts about 40 percent. Note that most of the deviation of the dotted line from the long straight line occurs for $M < 0.2$. Therefore, if we assume that at $M = 0.2$ the lines become unsaturated, the extinction then follows the short solid line to $M = 0$, rather than the dotted one; in this case, the true intensity outside the atmosphere is between those predicted from the square-root law and Beer's law, but nearer to Beer's law.

Figure 1 is constructed on the assumption that all of the extinction is due to saturated lines and bands. If we now assume that one half of the differential extinction between $M = 1$ and $M = 2$ is wavelength independent, such as that due to dust, Figures 2 and 3 are the results. As before, the extinction coefficient is such that $I\,(M = 1)\,/\,I\,(M = 2) = 1.2$. If further, we assume that the lines become unsaturated at $M = 0.2$, the error of the Beer's law extrapolation to $M = 0$ is about 4 percent, while the error of the square-root method is about 12 percent. Note,

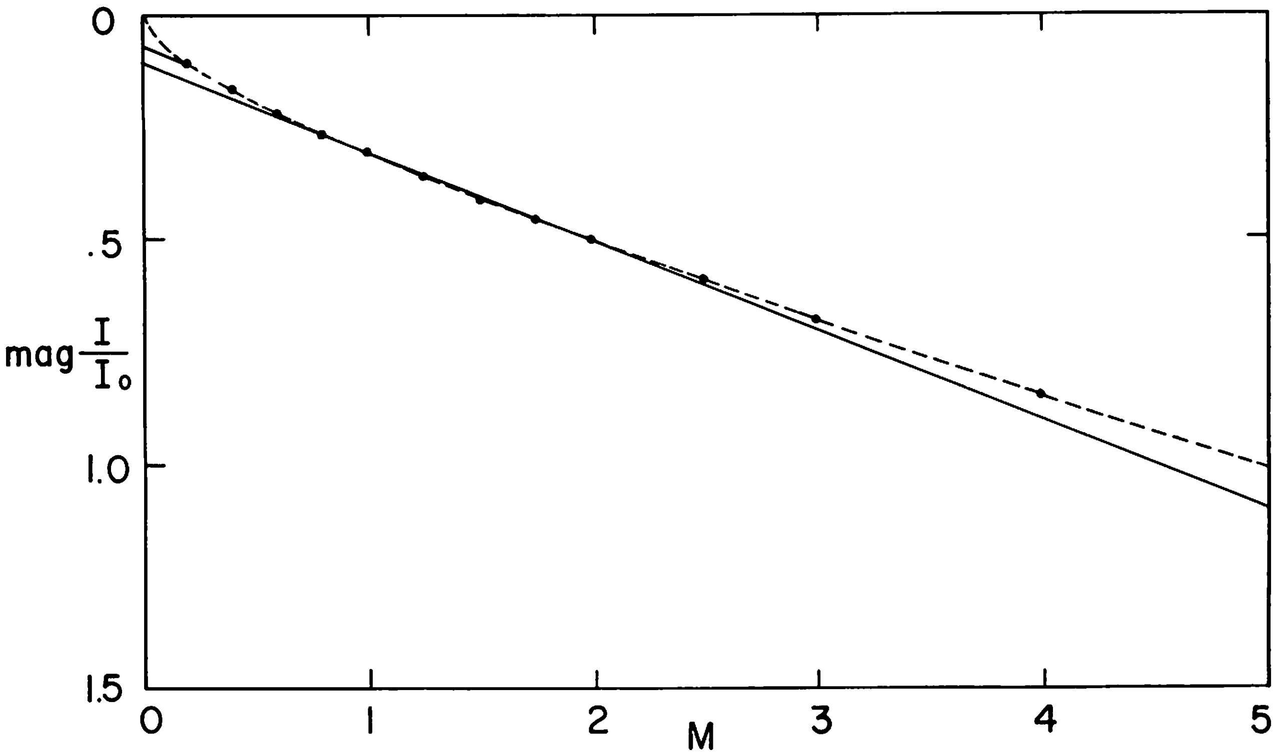

Fig. 2 Extinction versus air mass, M. Symbols are the same as for Fig. 1. It is assumed that half of the extinction is wavelength independent.

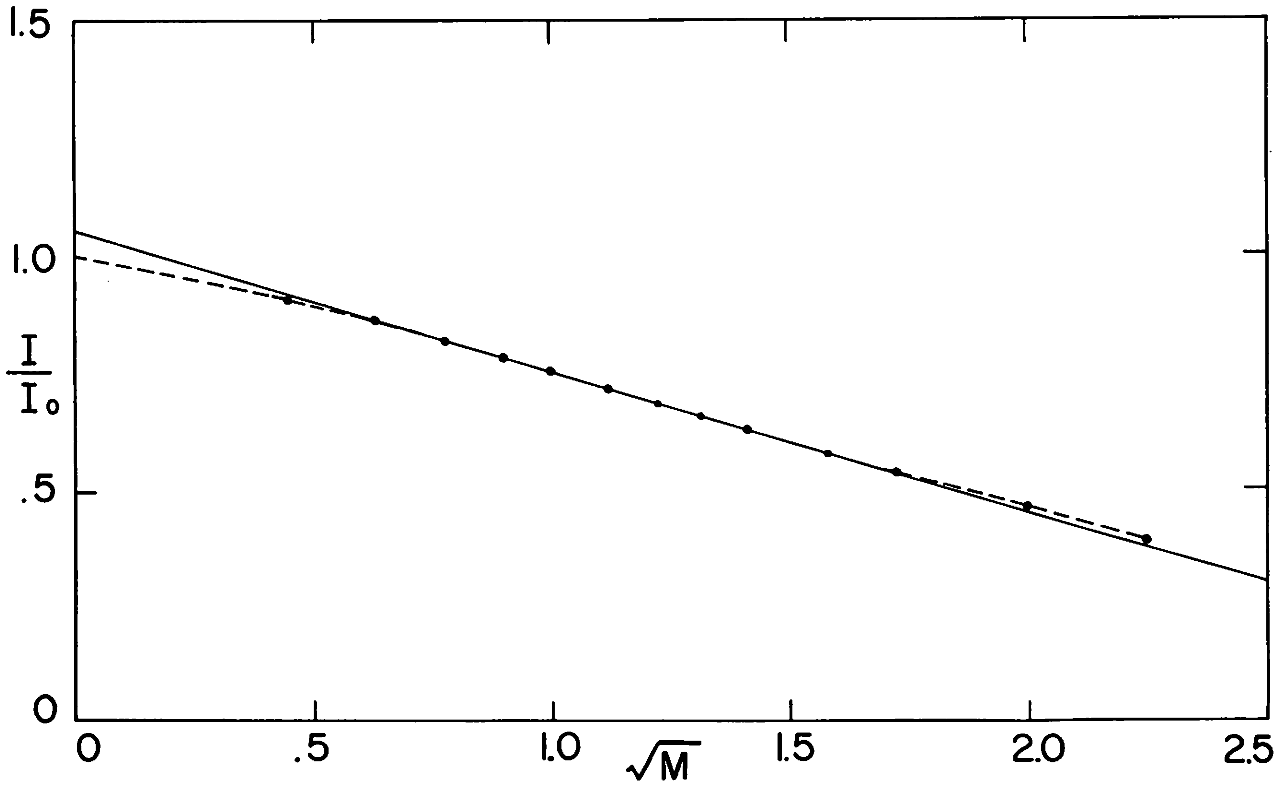

Fig. 3 Relative intensity versus the square root of the air mass. Symbols are the same as for Fig. 1.

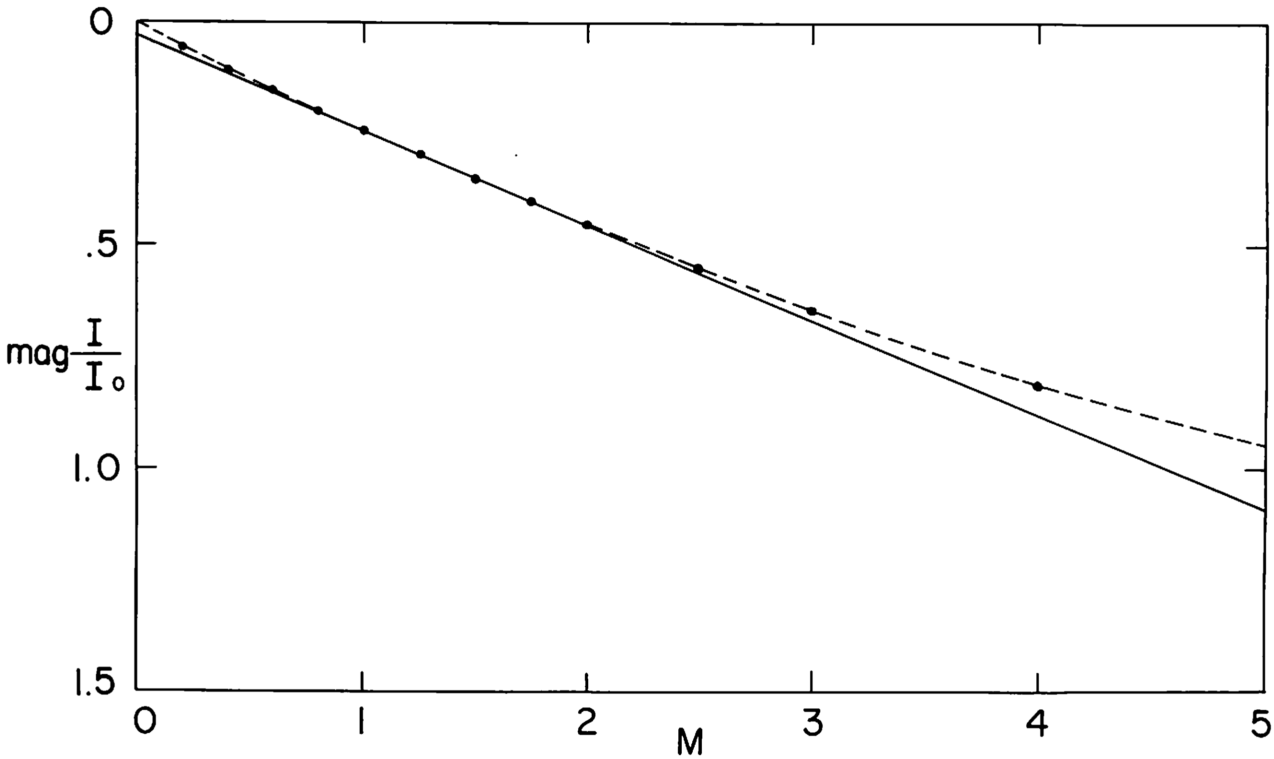

Fig. 5 Extinction versus air mass, M, for the artificial line. Symbols are the same as for Fig. 1.

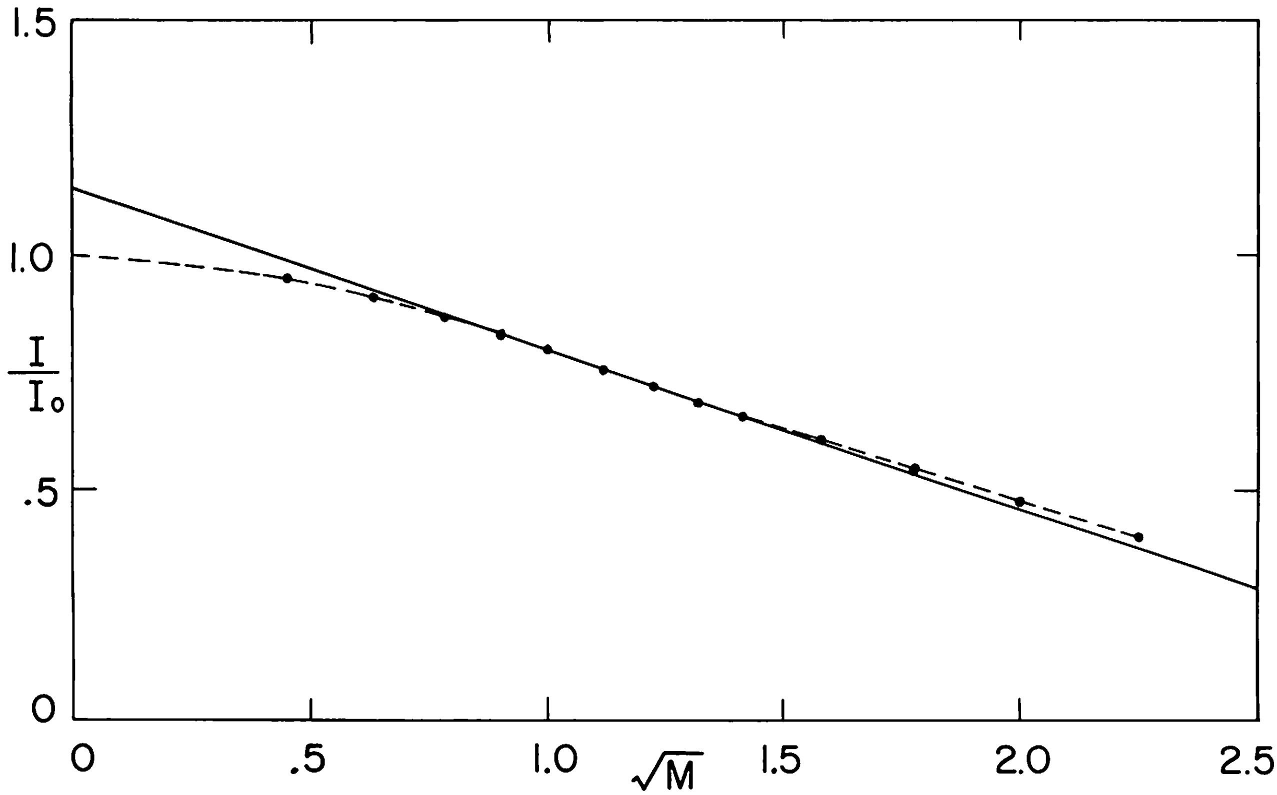

Fig. 6 Relative intensity versus the square root of the air mass, M, for the artificial line. Symbols are the same as for Fig. 1.

however, that for large air masses, the square-root law is more accurate. These assumptions, that the extinction due to saturated lines and from other sources are about equal at $M = 1$ or 2, and that the saturated lines become unsaturated for $M < 0.2$, seem fairly realistic since in our actual observational procedure we deliberately avoid the spectral regions in which the strong atmospheric absorptions occur.

It has been said, in defense of the use of the square-root law for outside-atmosphere extrapolations, that while we do avoid the regions of the saturated lines, we are still working in the wings of these strong saturated lines, the implication being that the square-root law applies in these wings. However, Equation (5) applies to the *entire* line and it does not follow that the absorption of a section of the wing also obeys this equation; in fact, it does not.

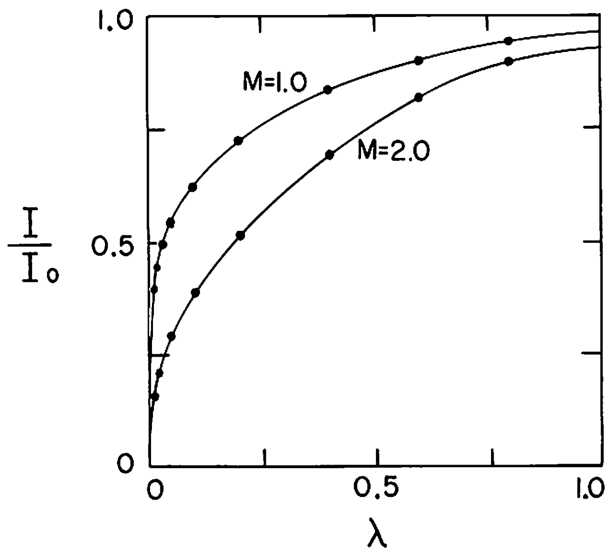

Fig. 4 The profiles of the artificial absorption line.

To illustrate this point, we assume in Equation (4) that

$$K(\lambda) = -\log \lambda + \text{const}, \; 0 < \lambda < 1. \qquad (7)$$

The filter band transmission is assumed to be unity, $0 < \lambda < 1$, and zero elsewhere. The variations of I/I_0 with λ, for $M = 1$ and 2 are shown in Figure 4. The resulting extinction curves are shown in Figures 5 and 6. It is clear that, while the square-root law (Fig. 6) fits much better for large air masses, the Beer's law extrapolation to $M = 0$ is nearer to the true value. For this case, the errors are:

Beer's law 3%,
Square-root law .... 14%.

4. Conclusions

These computations indicate that the use of the square-root law for extrapolation of observed intensities outside the atmosphere will in all practical cases lead to energies that are larger than the true values. They also indicate that fairly accurate ($\pm$ 2 percent or so) extrapolations can be made by using Beer's law with the extinction coefficient determined from observations at $M \sim 1$ and ~ 2, and then increasing the extrapolated value by 15–30 percent of the Beer's law coefficient.

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